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SOME PROBLEMS OF SIDEREAL ASTRONOMY*

BY HENRY NORRIS RUSSELL

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The main object of astronomy, as of all science, is not the collection of facts, but the development, on the basis of collected facts, of satisfactory theories regarding the nature, mutual relations, and probable history and evolution of the objects of study. Before the existing data appear sufficient to justify the attempt to form such a general theory, two policies of investigation may be followed: (1) to collect masses of information, as accurate and extensive as possible, by well tested routine methods, and leave it to the insight of some fortunate and future investigator to derive from the accumulated facts the information which they contain regarding the general problems of the science; (2) to keep these greater problems continually in mind, and to plan the program of observation in such a way as to secure as soon as practicable data which bear directly upon definite phases of these problems.

Much valuable and self-sacrificing work has been done by astronomers who adopted the former policy. In the opinion of many investigators, however, the progress of astronomy would be hastened if fuller consideration were given to the second method of attack, especially with a view to the widest possible coöperation between different observers and institutions.

In the hope that the committees of the National Research Council may be of service in furthering such coöperation, and at the request of the Chairman of the Council, the following survey has been attempted of the general problems of sidereal astronomy, and of investigations which at present promise advances toward their solution.

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I. The Individual Stars.—Existing methods of investigation have already put at our disposal a great mass of information regarding the physical characteristics of the stars—mass, density, luminosity, color, spectrum, temperature, and so on. The central problem of stellar astronomy may be formulated as follows: From the existing data, and from all further data which may be secured by methods new or old, to deduce a theory of stellar evolution, that is, of the changes in the temperature, density, brightness, spectrum, and other observable characteristics of a star with the progress of time, and of the dependence of these changes upon those factors which are invariant for a given system, such as mass, angular momentum and chemical composition. Such a theory must satisfactorily represent the observed properties of the general run of the stars, and the relative abundance of the different types, and must be capable of extension so as to account for the exceptions to the usual rules.

Among the subsidiary problems whose solution is bound up with that of the main problem are (*a*) that of the evolution of binary systems, whether by fission, tidal action, or otherwise; (*b*) that of the causes and mechanism of variable brightness among the stars; (*c*) that of the source of the energy which the stars radiate into space in such enormous amounts.

These problems of stellar astronomy are mainly physical in character, though some phases, such as (*a*) are mainly dynamical.

II. The Galactic System.—The great majority, if not all, of the visible stars appear to belong to an assemblage limited in space, either by regions nearly void of stars, or by absorbing material which conceals whatever may be immersed in it. Within this galactic system we may investigate the distribution of the stars in space, and its variation for stars of different spectral type, absolute magnitude, etc., the motions of the stars (including the Sun), and the phenomena of preferential motion ('star-streaming') in certain directions, and the dependence of these motions upon spectral type, absolute magnitude, etc.; and the association of the stars into sub-groups, or clusters, and the motions of these clusters.

All these studies lead up to a single ultimate problem, which may be defined as the representation of the present positions and motions of the stars as a stage in the history of a dynamical system (whether in a steady state or not) and the deduction of the presumable history of the system in the past and the future. Among the subsidiary problems connected with this are (*a*) the existence, character, distribution and gravitational influence of possible dark or absorbing matter in space; (*b*) the

relation between the age or evolutionary stage of a star and its position and motion within the galactic systems. The latter connects the problems of stellar and galactic evolution in such a way that any notable advance in the solution of one is likely to be of aid in that of the other, while an unfounded assumption regarding either will probably confuse the discussion of both.

III. Clusters and Nebulae.—So little has been known of these objects until very recently that the problems which they present can hardly yet be coördinated into a single statement. Among the most obvious are:

(1) The relations of clusters and nebulae to the galactic system. It now appears probable that the galactic system is very much larger than was supposed a few years ago, and that not only the irregular clusters, and the gaseous nebulae, both planetary and extended, but also the globular clusters, and probably the Magellanic Clouds, lie within its confines. But the relations of the spiral nebulae are still uncertain.

(2) Motions and dynamical relations within clusters, especially globular clusters, and explanation of the law of distribution of stars in such clusters.

(3) Nature of the gaseous nebulae, especially of 'nebulium,' and cause of their luminosity. Internal motions in gaseous nebulae.

(4) Nature of spiral nebulae, and explanation of the rapid motions of their parts.

In all these cases a persistent attempt should be made to account for the observed phenomena by means of the known properties of matter and forces of nature, and the existence of unknown forces should be postulated only if there is apparently no escape from the necessity of doing so.

It may now be profitable to survey rapidly the different fields of astronomical investigation, and consider the bearing of various researches —some now under way, some practicable at present, and others desirable if means for effecting them can be devised—upon these general problems.

1. Spectra.—It seems to be increasingly clear that the master-key to these problems, so far as they have yet been formulated, lies in the investigation of the spectra of the stars and other bodies, and the correlation of their other characteristics with the spectra. Fortunately, the spectra are among the few characteristics which can be investigated independently of any knowledge of the distances of the various bodies,— and, indeed, of the distances themselves, except for the limitation arising from the faintness of most of the remoter objects.

(*a*) Two fundamental facts appear upon the study of the lines of stellar spectra. The first is that almost all of the thousands of lines which have been observed are identifiable as those of known elements, and can be reproduced under conditions which can be realized in terrestrial laboratories. The few outstanding exceptions are yielding year by year. The recent identification of the G band in the solar spectrum as due to hydrocarbons,[1] and of the bands of ammonia[2] and water-vapor[3] in the ultra-violet, leaves very few 'unknown' solar lines of any importance. Nor are there any of great account in stellar spectra, except in stars of the fourth type (Class N) and in the Wolf-Rayet and 'early' helium stars.

So many of the lines in the latter have recently been found to be identical with those given in the laboratory by familiar elements (such as hydrogen, oxygen, carbon, and helium), under unusually intense electrical excitation[4] that there is good reason to hope that further researches in this direction may account for those which still remain, and even solve the long-standing riddle of the origin of the characteristic nebular lines (which are associated with the Wolf-Rayet lines in the nuclei of planetary nebulae and in new stars at certain stages). The spectrum of the solar corona, however, still remains an isolated problem.

(*b*) The second great fact is that the vast majority of stellar spectra fall into a single, continuous, linear sequence, which forms the basis of the Harvard system of classification, now generally adopted. Almost all the spectra which did not obviously belong to this sequence have been brought into connection with it by the recent work of Wright,[5] connecting the gaseous nebulae with the Wolf-Rayet stars at the head of the series, and that of Curtiss and Rufus,[6] which shows that the small but definite classes R and N form a sort of side-chain, branching from the main sequence near the other end, at class G (or perhaps K). Miss Cannon's experience[7] in classifying over 200,000 spectra shows that the objects that do not fall into the sequence, thus extended, are almost vanishingly rare.

The general characteristics of this sequence are now well established, and the types which were selected, by a sort of survival of the fittest, in the evolution of the Harvard classification prove to have been surprisingly well distributed along the series. With the aid of the quantitative methods of classification developed by Adams and Kohlschütter,[8] the precise classification of any spectrum of which a good photograph with suitable dispersion is available should be an easy

matter, even in the interval between G and K5, where the differences between consecutive types are least prominent. The publication of a detailed descriptive 'key' with good reproductions of photographs of spectra of each successive class would however be a great boon to isolated workers.

Of much greater importance is the devising of some method for photographing the spectra of stars fainter than the tenth magnitude—which are now about at the limit of accessibility. Long exposures with the objective prism are greatly embarrassed by difficulties in guiding, but the problem is doubtless soluble in some way, and ought to be solved.

(*c*) There is now good reason to believe that the differences between the main classes of spectra arise from differences in the effective surface temperatures of the stars, and that differences in their other physical characteristics play only a minor rôle in the spectra, but reveal themselves in differences in detail, formerly described as 'peculiarities' when they were noticed at all. The investigation of these finer differences is to-day the most promising field in stellar spectroscopy.

What valuable results may be obtained was shown by Hertzsprung's[9] work on Miss Maury's 'c-stars' (with unusually sharp spectral lines) which prove to be of greater real brightness than any other class of stars so far known; and later, and still more remarkably, by Adams' and Kohlschütter's[10] discovery that the absolute magnitudes of stars (of the 'later' spectral classes, at least) can be predicted with surprising accuracy from the relative intensity of a few pairs of lines in their spectra. The data for stars of great luminosity are still scanty, but should be easily obtainable, using the hundreds of spectrograms now available at the great observatories, and determining the mean absolute magnitude of groups of stars, which the spectroscopic method indicates as being of similar brightness, by means of their parallactic motions. When this has been done, our knowledge of the distribution of the naked-eye stars in space will be very greatly advanced.

The careful comparison of the spectra of pairs of stars otherwise similar, but known to differ in other characteristics than absolute magnitude, may yield results of importance. Many recognizable spectral 'peculiarities' too, such as the diffuseness or sharpness of the lines, the presence of bright lines, the abnormal strength or weakness of certain lines, etc., have as yet been very incompletely studied, especially as regards their relation to other characteristics of the stars. For example, it should be possible to distinguish between widening of spectral lines due to a star's rotation, (which would affect all lines alike), and

widening due to physical conditions in its atmosphere (which are likely to affect some lines more than others).

(*d*) Another promising field is found among the reddest stars. Curtiss makes the very interesting suggestion that the division of the spectral series into the branches G–K–M and G–R–N (or perhaps K–R–N) may be due to differences of chemical composition[11]—since it is known that the surface temperatures of these stars are low enough to permit the formation of chemical compounds. If this is true, the strength of the characteristic bands of titanium oxide or of carbon should depend upon the relative proportions of these elements, and show little correlation with the color index, or the extension of the spectrum in the violet, which depend primarily on the temperatures. There is already considerable evidence that this is actually the case, and it may be remarked that the star Epsilon Geminorum, which is of spectral class G5 has a color index (+1.52) almost equal to that of Classes M or R.[12] This star may be in the situation anticipated by Curtiss, in which an exact chemical equilibrium between carbon and titanium oxide suppresses the bands of both.

Photography of the spectra of bright stars in the red, and even the near infra-red, is now practicable, and Merrill[13] has already obtained results of great interest and promise. Investigation of the spectra of the brightest stars with high dispersion is also profitable, as is shown by the work of Adams[14] upon the pressures which probably prevail in the atmospheres of Sirius, Procyon, and Arcturus. Fortunately, the stars brighter than the second magnitude afford examples both of giant and dwarf stars of almost every spectral class.

2. (*a*) Almost equal in importance to the line absorption in stellar spectra is *the distribution of intensity in the continuous background*. The most complete and satisfactory method of studying this would be the direct measurement of the energy carried by different wave-lengths, but this has not yet been proved practicable. A first step has however been taken by Coblentz,[15] who has not only measured the total energy radiation of more than a hundred stars, but in some cases the percentage transmitted by a water cell, thus providing our first knowledge of stellar radiation in the infra-red. With the great reflectors just completed, the determination of spectral energy curves for the brightest stars may be possible.

The distribution of energy in the luminous region of the spectrum is however readily determinable. For the brighter stars, spectro-photometric methods can be employed, as in the visual work of Wilsing

and Scheiner,[16] and the photographic investigations of Rosenberg.[17] Fainter stars, down to the sixteenth magnitude, at least, can be reached by the determination of *color indices*.

(*b*) In order that these color indices may be capable of full utilization, it is necessary, first, that trustworthy and homogeneous scales of visual, photographic and photovisual magnitudes be established over the whole range of about 47 magnitudes from the Sun to the faintest observable stars. This problem, which is fundamental in all stellar photometry, is already well advanced toward solution. But in the second place, it is necessary that the physical meaning of the units of magnitude should be precisely known; that is, that the 'luminosity curve' which expresses the relative sensitiveness of the photometric receiver for equal energy of different wave-lengths should be exactly determined. And, above all, it is essential that this luminosity curve should be independent of the brightness of the stars under observation. These last two conditions are at present very imperfectly satisfied, if at all. Very little is known about the luminosity curves of the standard plates and apparatus used in the determination of photographic and photovisual magnitudes, and nothing at all about the luminosity curves of the eyes of the 'standard observers' at different observatories,—except that they must be very different under the conditions prevailing at Harvard and at Potsdam.[18] It is certain that the Purkinje effect alters the form of the visual luminosity curve as the brightness of the illumination varies, probable that this affects the visual comparison of the brightness of stars of widely different magnitudes, and uncertain whether, and to how great an extent, similar photographic influences exist.[19]

The direct determination of the luminosity curves for the principal instruments and methods employed in the determination of photographic and photovisual magnitudes would be neither difficult nor laborious. For visual observations it can be derived indirectly, if direct measures prove difficult. To make these investigations at once is urgently desirable, for the present bases of the scales of stellar magnitude are not permanent. The photographic and photovisual scales depend on the properties of present commercial types of rapid plates, which may not be manufactured a few years hence if improvements are devised; and the visual scales are based on the characteristics of the eyes of observers some of whom have already retired from active work.

Such an investigation would also establish a connection between the scales of stellar magnitude and the physical units of measurement of light in the laboratory (which are now defined in terms of a definite

luminosity curve), and would enable us to express our stellar photometric data in absolute units.

It is also desirable that methods for measuring the brightness of the stars with red and ultra-violet light should be developed, with careful determination of the luminosity curve in each case, and of the color equation which (for normal stars) makes it possible to reduce color-indices obtained on any of these systems to a standard scale.

The determination of the colors of faint stars by other methods affords a promising field, as is shown by the success of the method of effective wave-lengths,[20] and of that of exposure ratios,[21] recently developed at Mount Wilson.

Such a determination of exact scales of magnitude and color index is evidently a necessary condition for the full utilization of the great mass of material which is in process of collection concerning the numbers of stars of different magnitudes, their concentration towards the Galaxy, etc.

(*c*) The statistical investigation of the relations between color index and spectral type, and between both and absolute magnitude, have already opened up possibilities of estimating the distances of stars far too remote to be reached in any other way. Such investigations should be extended, with special reference to stars of great and small absolute brightness, and to those having peculiar spectra.

Closely connected with this is the question of possible selective absorption of light in space. Shapley's results,[22] and the theoretical work of L. V. King,[23] appear to negative the existence of any general absorption of this sort. But local selective absorption may occur, and it would be well worth while to study intensively the color indices and spectra of stars in regions where the existence of absorbing matter is suspected, such as Barnard's dark lanes in Scorpius. It is interesting in this connection to note that the three most abnormally yellow stars of Class B (ζ, o and ξ Persei)[24] lie within 5° of one another, in a region full of diffuse nebulosity.[25] A survey of the stars in this region for color-index and spectral type would be well worth while.

(*d*) Another interesting problem is presented by the extreme infrequency of very red stars. Color-indices up to about +1.8 on the Harvard scale are fairly common; but greater values are very unusual, and are practically confined to the 'side chain' which includes Class N. In this subsidiary sequence the color-indices increase to about +4, as might be expected as a result of decreasing temperature; but in the main series, ending in Class M, this does not happen. Are all the stars of Class M of about the same temperature, or is an increase of redness in Classes Mb and Mc masked by increasing absorption in the

red end of the spectrum? There are certainly very heavy absorption bands in the red in these spectra; and further evidence in favor of this hypothesis is found in Coblentz's measures of Alpha Herculis,[26] which show this star, of Class Mc, radiates far more heat in proportion to its light than do stars of Class Ma, and also in Hertzsprung's[27] observation that the very faint dwarf stars of Class Mb are not nearly as red as their small luminosity, and probable low surface brightness, would lead one to suppose. A careful study of the color indices, and, if possible, of the spectral energy curves, of the stars of Classes Ma, Mb, and Mc is much to be desired. The extraordinarily red stars S Cephei[28] and +43°53,[29] which have color indices exceeding five magnitudes, should be included in such a study.

3. One other stellar characteristic which may be investigated without knowledge of distance is *variability of brightness*. If we really understood the causes of stellar variability, we should probably have advanced a long way towards the solution of the whole problem of stellar evolution, if not have solved it completely. But, in spite of the great number of variable stars, the variety of the phenomena which they represent, and the accuracy with which they can now be observed, the humiliating admission must be made that no even tolerably satisfactory theory of the causes of the variation exists, except for the eclipsing variables, and in this case it is based on the proposition that, except for the accident of eclipse, the components are not variable at all!

Successful attack upon the problem of intrinsic stellar variation will probably demand the correlation of all the data that can be brought together from every accessible source. In the case of regular variables, precise light curves are of importance, and many stars still await investigation,—some of them visible to the naked eye, and long known to be variable. The new photometric methods of precision—especially the photoelectric cell—have opened a wide field in the study of bright stars with small variation, in which important results have already been obtained,—notably by Stebbins[30] and Guthnick,[31]—and more may be anticipated.

(*a*) Former suspicions of changes in form of the light curves appear to have been unfounded in the case of eclipsing variables; but similar changes are believed with good reason to exist among Cepheid variables.[32] To prove their reality—still more to discover their laws—demands very precise observations, preferably by two or more observers at different places and the same time.

(*b*) Changes in color, as well as in brightness, appear to be the general—perhaps the invariable, rule among eclipsing variables, and especially

among Cepheids—the star being always redder at minimum than at maximum. More recent observations show that changes in the spectrum go hand in hand with the others.

In the case of eclipsing variables, these changes arise from a difference in spectral type between the components, and it is found that stars separated by an interval less than their own diameters, and therefore very probably of the same origin and age, may have spectra differing as widely as those of Sirius and Arcturus.[33] Observations of such systems, when the eclipse is total, provide the only direct method at present existing for studying the relations between spectral type, color index, surface brightness, and density, which are of fundamental importance. The determination of the spectral type of the fainter components of such systems, though often very difficult, on account of their extreme faintness, deserves special effort.

(*c*) The concomitant variations in brightness, color, and spectrum, which Shapley[34] has shown to occur in every Cepheid variable that has been properly investigated, indicate very strongly that the proximate cause of the changes in all three is a periodic variation in the surface temperature of the stars. Shapley's suggestion[35] that these differences in temperature arise from some sort of internal changes, perhaps of the nature of periodic oscillations in the radius, density, temperature, etc., appears to be the best which has been yet made; but there are still grave difficulties in explaining how such pulsations should in all cases produce the very distinctive form of the light curve, with its rapid rise and slow fall, and still greater trouble in accounting for the variations in radial velocity, which show so remarkable a relation, both in amplitude and phase, to those in light. It is in fact still doubtful whether these stars are really binary systems or not. Intensive studies of a number of these variables, including the greatest practicable variety of representative cases, would be well worth while.

(*d*) Still less is known concerning the very numerous variables of long period, and the roughly periodic and irregular variables. In the observation of their changes in brightness, amateur observers may obtain results of much value, and, under the admirable coöperative schemes organized by the American and British Associations of Variable Star Observers, they are at present furnishing a great mass of valuable information. Very little is known regarding changes in the spectra of long-period variables, except that they often exist,[36] especially as regards the bright hydrogen lines which are usually present at maximum. Observations of the color indices of these variables are also much to be desired. Certain peculiar variables, such as R Coronae and SS Cygni,

are typical of small but definite groups, whose variation, though quite distinctive, is entirely unpredictable. The spectra of the stars of the first of these groups are similar to one another, and unlike anything else.[36] Those of the second group are also peculiar, and appear to be variable.[36] Both present problems as alluring as they are difficult. The spectra of other peculiar variables also deserve investigation.

(*e*) New stars are usually pretty fully observed while they remain bright, but work remains to be done in following at shorter intervals the changes during their later stages. The recent work of Adams and Pease[37] indicates that they settle down into Wolf-Rayet stars; but, according to Miss Cannon,[38] the spectrum of the Nova in Corona, fifty years after its outburst, is now of class K. No one seems yet to have followed up Hertzsprung's interesting suggestion[39] that stars of very small absolute luminosity should be investigated for variability. Abundant material for a photographic study must exist in the Harvard collection.

4. Knowledge of the *distances* of the stars is indispensable in the solution of many problems. The nearer ones, to a distance of thirty parsecs or so, are now accessible to direct measures of *parallax*, and great activity prevails in photographic observation for this purpose, in accordance with a wide and well-considered plan of coöperation.

In my opinion, however, the greatest need in parallax work at present is the investigation and elimination of the systematic errors which are still present in the best work, as is shown by the too frequent appearance of large discordances—sometimes amounting to more than $0''.05$—between the results of different observers, although the probable errors derived from the internal agreement of each observer's plates are of the order of $\pm 0''.01$. The intercomparison of the results of various observers for the same stars is hardly a sufficient test for the absence of systematic error, especially as all are using nearly the same method of observation. The only secure control is afforded by observing stars whose parallaxes can be predicted, from other considerations, with greater accuracy than they can be observed. This demands prediction with a probable error not exceeding $\pm 0''.005$. Fortunately, several groups of stars exist for which such prediction is possible. The most prominent of these consists of those stars of spectrum B which are between 60° and 120° from the solar apex. If the parallaxes of these stars are computed on the assumption that their individual proper motions are entirely due to the solar motion, the resulting errors will correspond to a probable error of less than one-third of the parallaxes themselves—that is, to about $\pm 0''.002$. The

stars of Kapteyn's Scorpius-Centaurus group[40] would be ideal objects, if they were not too far south.

For fainter stars, eclipsing and Cepheid variables are available. Of the 90 eclipsing variables whose parallaxes were estimated by Russell and Shapley,[41] 69 are fainter than the eighth magnitude, and the mean parallax of these is 0".002, while only ten per cent exceed 0".004. For the Cepheids of similar brightness, the parallaxes estimated by Hertzsprung and Shapley[42] are even smaller.

When once the systematic errors have been tracked to their source and eliminated, an extensive program of observation can be undertaken with security. Much duplication of observations is desirable, for it is obviously better that the parallax of a star should be determined from the mean of three or four short series of as many different observatories than by a series with a single instrument, however long and elaborate. Certain objects for which especially accurate parallaxes are desirable should be observed at as many places as possible. Examples are binary stars, stars differing in absolute magnitude from the bulk of those of the same spectral class, or from the values predicted by the spectroscopic method, stars with exceptionally rapid motions in space, planetary nebulae, etc. Attempts to determine by direct observation the mean difference in parallax between classes of stars with small parallaxes (for example, those of the third and fourth magnitudes, taken as a whole) should, in my judgment, be deferred until the systematic errors have been thoroughly cleaned out.

5. Knowledge of parallax leads at once to that of *absolute magnitude*, which, in the interest and importance of its systematic relations to other characteristics of the stars, stands second only to spectral type.

(*a*) The relations between the two afford a very interesting study, which has led Hertzsprung[43] and Russell[44] to the recognition of the two series of 'giant' and 'dwarf' stars, coincident in class B, but gradually drawing apart among the redder stars until, as Adams' spectroscopic results have recently confirmed,[45] they are completely and widely separated in class M. If Russell's views are correct, the existence of these two series is the key to the problem of stellar evolution. In any case, their existence must be accounted for, and will be of importance in testing any theory. The securing of additional data, especially regarding the absolute magnitudes of individual giant stars, is much to be desired. It is of importance to determine not only the mean absolute magnitude of the giant and dwarf stars of each spectral class (whenever the two are separated) but the dispersion of the individual values about the mean. Only when the latter is known can the results

of statistical investigations be cleared from the effects of the egregious observational preference for the brighter and remoter stars.

(*b*) Kapteyn[46] has obtained fairly good values of the dispersion among the various divisions of Class B, and provisional values for Class A; and Russell[47] has given rough estimates for the dwarf stars, and a still rougher one for the giants of Class M: but further work is greatly needed. Adams' spectroscopic method offers an easy solution of the problem, as soon as his present provisional scale of absolute magnitudes for the giant stars has been revised with the aid of studies of the parallactic and peculiar motions of groups of stars whose spectra indicate that they are similar in real brightness. Strömberg[48] has already shown in this way that Adams' mean absolute magnitude for all the giant stars, taken together, is substantially correct; but there is evidence that the provisional estimates for the very brightest stars (such as the Cepheid variables) make them considerably too faint.[49]

(*c*) The existing evidence indicates that the majority of the stars of any given spectral class are confined within surprisingly narrow limits of absolute magnitude (provided that the giants and dwarfs can be treated separately). But there are exceptions of great interest. For example, Kapteyn[50] has shown that β Orionis is some eight magnitudes brighter than the average for its class (B8); and the faint companions of Sirius[51] and o^2 Eridani[52] have spectra of class A, although they are at least eight magnitudes fainter than normal stars of this class. Exceptional brightness is probably explicable by unusual size or mass; but the two exceptionally faint stars (which are known to be of normal mass for stars of their brightness) present a real puzzle. Something about the physical conditions in these stars must be very unusual, and they should be studied with the greatest attainable detail. Other such objects may be found among the faint stars of large proper motion.[95]

6. Beyond the limit of direct measures of parallax, our main reliance must be placed on *proper motions*, which are of fundamental importance in the study of the galactic system.

The brighter stars have already been cared for by Boss, and those down to magnitude 7.5 are under discussion. The fainter stars can best be investigated by photography, carrying the work to objects as faint as can be reached with large instruments, in accordance with Kapteyn's 'Plan of Selected Areas' or some equivalent. For this purpose, it is essential to have a set of reference stars, distributed uniformly over the sky, and of suitable brightness to serve as photographic standards, and to make the observations strictly differential with respect to these, using them not merely as reference points for position when reducing a

single plate, but as reference points for proper motion when comparing two plates of different epochs. The observations of these reference stars must at present be made with meridian circles; but the proposed methods for determination of absolute positions of the stars by photography deserve careful study and trial.

Pending the completion of such a program, the investigation of the proper motions of faint 'optical' companions of bright stars, such as has been made by Comstock,[53] furnishes our best source of information concerning the proper motions of faint stars, but is complicated by systematic errors in the early measures. A survey of the whole heavens for stars of large proper motion is very desirable. In this case it is legitimate to treat the general 'background' of stars as at rest, and the observations can be very rapidly made, with the blink microscope or similar appliances. Early plates are probably already available for almost, if not quite, the whole of the heavens. Such an investigation is likely to yield important information concerning the stars of very small absolute luminosity—as is shown by Barnard's[54] and Innes's[55] recent remarkable discoveries—and should be extended to the faintest accessible stars.

Comparison of measures of plates taken at different epochs (still treating the bulk of the stars as fixed) will yield much information about proper motions of moderate size. This has already been done on an extensive scale with plates of the Astrographic Catalogue.

Special investigations should be made to determine at an early date the proper motions of all stars belonging to certain interesting classes for which early determinations of position are available—for example, binaries, variables, and stars having peculiar spectra.

7. The study of the *radial velocities* of the stars is intimately associated with that of the proper motions. The determination of radial velocities with the slit spectroscope has been brought to a high degree of perfection, but the separate investigation of each one of the many thousands of stars which are now accessible would involve an enormous amount of labor. The development of some method by which radial velocities could be determined *en masse* with the objective prism would be a great boon. If some absorbing medium giving sharp and well distributed lines in the blue and violet could be found, the problem would become simple; and other solutions are doubtless possible.

It is also desirable that some method be devised for obtaining, at least approximately, the radial velocities of stars possessing spectra with very diffuse lines. At the present time, no radial velocities have been published for some of the very brightest stars, on this account.

In extending the list of observed radial velocities, much advantage has been gained by a policy of selective observation of classes of stars of special interest—such as stars of unusually large and small proper motion, absolute magnitude, and the like, variable stars, and stars of the rarer spectral types. A similar investigation of double stars showing evidence of physical connection would be worth while.

8. Statistical discussions of the motions of the stars and of the Sun, and their relation to spectral type, etc., offer an extensive and very intricate field. Among the matters demanding further investigation may be mentioned the reason for the differences in the direction and velocity of the solar motion derived from stars of different spectral types, and from proper motions and radial velocities separately; the origin of the constant term in radial velocities (Campbell's K term); the existence of tendencies toward common motion among the stars in particular regions of the sky; the dependence of the mean peculiar velocities of the stars upon spectral type and absolute magnitude, and the real cause of this dependence (possibly a correlation between large velocity and small mass); the true nature of preferential motion, and whether it really gives evidence of the existence of two physically different 'streams;' the dependence of preferential motion upon spectral type, absolute magnitude (the latter an unworked field) and perhaps upon the region of the sky considered; the devising of a rapid method for the detection of moving clusters, and the identification of their members; and so on. The discussion of most of these problems should be based simultaneously on proper motions and radial velocities. Results derived from either one alone may fall into errors which the combination of both would detect.

One practical matter deserves specific mention. When it appears desirable to exclude certain stars from a statistical discussion (for example, those of very large proper motion), the limits of exclusion should be clearly and precisely stated. Neglect to do so may cause great trouble to other workers who wish to make a comparison with their own results, and has sometimes led to very serious errors of interpretation.

9. Another set of data of fundamental importance depend upon relations involving the *masses* of the stars. Here there appears the grave difficulty that nothing at all can at present be found out concerning the mass of a star unless it is double. There are plenty of double stars, to be sure; but what certainty have we that they are similar in mass to stars which are not double? Only an indirect answer is possible, by means of the statistical comparison of single and double stars with respect to as many characteristics as may be—absolute magnitude,

spectrum, color, radial velocity, proper motion, distribution in space, etc. (bearing in mind that the limits of telescopic resolution restrict our knowledge of the remoter pairs). But Eddington's recent theoretical researches[82] lead to the hope that it may some day be possible to estimate the mass of any star when its absolute magnitude and spectral type are accurately known (using the data for double stars as a guide).

(*a*) As regards the determination of the masses of individual stars, it should be borne in mind that, for statistical purposes, a pair in which the relative motion of the components is known, though the motion in angle may be only a few degrees, is very nearly as valuable as one which has completed a revolution—while a pair for which the relative motion is unknown is of no use at all. The slowly moving pairs which are often, but inaccurately, described as 'fixed,' possess an importance exactly analogous to the stars of small proper motion, and give us invaluable information about those stars which are bright in proportion to their mass—the giant stars, in fact. Now that the discovery of double stars is apparently in sight of completion, it is to be hoped that more attention may be given to the problem of determining the relative motion in as many systems as possible.

(*b*) The existing data suffice to show that the masses of the stars differ from one another less than any other of their characteristics—the whole range among well determined masses being from 20 times the Sun's mass to one-sixth of the Sun's, which may be compared with a range in luminosity of at least ten million fold. For this very reason, very careful observations are required to enable us to say with certainty that one star is more or less massive than another. It appears certain that the stars of spectrum B are unusually massive,[56] and there is sufficient evidence to show that, in general, stars of great luminosity are more massive than those of small absolute brightness, and that, among the dwarf stars, those of 'later' spectral type are of smaller average mass.[57] But there are very few cases in which we can be sure that a given star is more or less massive than the average for its type.

It is very desirable to determine how great is the range of difference among the masses of stars of similar spectral class or absolute magnitude. Extremely precise determinations of parallax will be needed if this problem is to be solved, but the effort will be well worth while. Sufficiently reliable values of the mean masses of stars of different groups have already been determined, to make it possible to estimate the parallaxes of all but the nearer binaries and 'physical pairs' more accurately than they can at present be observed.[57] This should be of aid in the interpretation of other statistical studies of double stars,

such as the proportion of double stars among all the stars of a given magnitude, the relative numbers of close and wide pairs, etc.

The determination of the relative masses of the components of binary systems will soon also be possible in many cases which have previously been somewhat neglected.

When a sufficient number of accurate determinations of mass have been made, a detailed study of the spectra of stars differing in mass should be made, in the hope of finding peculiarities depending directly on the mass, which might make it possible to estimate the masses of isolated stars.

(*c*) A great number of *spectroscopic binaries* await investigation, and more are continually being discovered. In the determination of orbits, preference should be given to those which show the spectra of both components, as it is only in this case that definite information can be obtained about the masses. Eclipsing and Cepheid variables are also worthy of special attention, and also stars of large proper motion, or others which appear to be dwarf stars.

It is very desirable that some method should be found for observing the spectrum of the secondary component when it is too faint to be directly seen. Perhaps Koch's spectromicrometer might furnish a solution. Favorable cases for trial, in which the brightness of the invisible secondary spectrum is known, may be found among eclipsing variables.

10. The *densities* of stars can so far be determined only when they are eclipsing variables. In this case, when both spectra can be photographed, the *diameters* of the components can also be found. Several systems of this sort, which have not yet been investigated spectrographically, are within the reach of existing instruments.

If, however, the relations between spectral type, color index, and surface brightness can be so well determined that it is possible to estimate the last of the three when the other two are known, it will then be possible to determine the densities of all visual binary stars, the linear diameters of all stars of known parallax, and the angular diameters of all the stars in the sky. The known eclipsing variables should afford sufficient material for a first investigation of the problem, if only sufficiently accurate information can be obtained regarding the color-equation of the visual and photographic methods of observation which have been employed at various observatories.

11. All that can be said at present regarding the *internal constitution* of the stars depends on Eddington's theoretical work,[82] which indicates that, in the stars of low density, the mass should be greatly condensed toward the center—the central density being 54 times the mean den-

sity. But the problem is capable of investigation by observation. There are many close eclipsing pairs in which the components are ellipsoidal in form, as is proved by variability of the Beta Lyrae type. In such systems the lines of apsides of the orbits should advance, at a rate depending on the masses, dimensions, and internal constitution of the components. If the last is like that of Jupiter or Saturn, the advance of periastron should be rapid. What little evidence there is indicates a slower motion, and hence a very strong central condensation; but more intensive studies are necessary before definite conclusions can be drawn. There are several systems for which the necessary data concerning the dimensions and forms of the orbits and the stars are accessible to suitably planned observations,—notably α Virginis and *U* Herculis. A careful study of such stars, by means of simultaneous photometric and spectroscopic observations, would be remunerative.

The singular and so far inexplicable changes which occur in the periods of most eclipsing variables, and so far have defied prediction, also deserve extended study; and Eddington has recently called attention to the fact that secular changes in the periods of Cepheid variables are likely to give a clue to the rate of stellar evolution.[58] The first scanty evidence points to a very extended time scale.

12. In the investigation of *star-clusters*, measures of position, for the purpose of detecting future proper motions, are obviously a duty to posterity. There is little chance that anything more than the motion of the clusters as a whole will be perceptible in our generation, and only measures of the utmost attainable accuracy and freedom from systematic error are likely to be of use to the astronomers of the future. Of far more promise are studies of the distribution of the stars within the clusters, their magnitudes, and, above all, their color indices. Such investigations, in Shapley's hands,[59] have given us for the first time a true conception of the distances and magnitudes of the globular clusters. Students of the subject are eagerly awaiting the detailed publication of the evidence on which he bases his conclusion that the apparent avoidance by these clusters of the region within 1500 parsecs of the galactic plane is due to a real absence of clusters from this region, and not to obscuration by absorbing matter.

The variable stars in clusters also deserve further attention. Those so far discovered appear to belong to the Cepheid type, which is natural, as these seem to be actually the brightest of all variables. Long period and eclipsing variables may yet be discovered among the fainter stars.

Good work can still be done also upon the irregular clusters,—as is shown by Trümpler's[60] study of the outlying members of the Pleiades.

One of the most attractive of unexplored fields is the investigation of the *Magellanic Clouds*. The small amount of work which has been done, mainly on the Smaller Cloud, has led to the discovery of a remarkable relation between the periods and absolute magnitudes of the variables in the Cloud,[61] to the estimate that its distance is 20,000 parsecs,[62] and to the discovery that the nebulae within it, and probably the Cloud as a whole, have a very high radial velocity.[63] The great instruments which are now being erected in the southern hemisphere may well be actively directed toward this region.

13. (*a*) Foremost among the many problems presented by the *gaseous nebulae* is the cause of their luminosity. In spite of our ignorance of the origin of the characteristic nebular lines, the appearance of such lines as λ 4686 in the spectra of nebulae, and of the Wolf-Rayet spectrum in their nuclei, suggests that in them "we are presented" (in Fowler's words)[64] "with phenomena which result either from the effects of powerful electrical actions or of very elevated temperatures." Though such conditions may easily enough exist in the nuclei, it is very hard to see how high temperatures can prevail throughout the whole volume of a nebula.* There are several possible ways out, however.

The electrical action may be a bombardment of the outer region by corpuscles emitted from the nucleus. Or perhaps the luminosity of the gases is fluorescent, like that of the sodium or bromine vapors studied by Wood.[66] Or, as Fabry has recently suggested,[67] we may have to do with a body which absorbs and emits radiation only in narrow regions of short wave-length, and may therefore attain a very high temperature in thermal equilibrium with the radiations from a distant, but still hotter, source. To determine the true explanation among these and many other possibilities may tax the resources of both experimental and theoretical spectroscopy.

The association of gaseous nebulae with stars of 'early' spectral type might be anticipated on any of these theories. For such stars are very hot bodies, and would be the most powerful sources both of corpuscular and ultra-violet radiation. Hence the association of these stars with nebulae does not prove that the stars originate from the nebulae. It is entirely conceivable that, on the contrary, the nebulae, as visible objects, owe their existence to the radiation of the stars, and are their offspring, and not their parents. Some gaseous nebulae, however, are not near bright stars, and the nuclei of planetary nebulae appear to be

* Fabry's calculated temperature of 15,000° for the Orion Nebula,[65] as he points out, is liable to be diminished by an unknown amount on account of the widening of the lines of the spectrum by turbulent motion of the nebular matter in the line of sight.

comparable with some of the faintest stars in luminosity. Clearly, nothing final can be said on this subject until we know what it is that shines in the gaseous nebulae, and why. It may be remarked, however, that the wide-spread assumption that the origin of the stars is to be sought in the visible nebulae appears to have had very little solid basis. All classes of nebulae except the extended gaseous nebulae have already been excluded from consideration as observational knowledge increased.

(*b*) A few nebulae, like those in the Pleiades,[68] appear to shine by light reflected from neighboring stars, and Slipher's spectroscopic work is steadily adding to the list of his discoveries in this field. Hertzsprung[69] has shown photometrically that the brightness of the nebulosity in the Pleiades is entirely consistent with the reflection hypothesis. Similar studies of other nebulae, and especially of the remarkable variable nebulae recently observed by Slipher,[70] would be of value.

Barnard's long continued researches[71] have made it highly probable that there exist many *dark nebulae*, revealed only by the effects of their opacity in concealing whatever lies beyond them. It is highly significant that the most remarkable of these dark regions is obviously directly connected with one of the nebulae which shines by reflected light,—that surrounding Rho Ophiuchi[72]—and that the whole mass is comparatively near us in space, at a distance of 100 to 150 parsecs. If such masses of practically opaque material are scattered through the galactic and extra-galactic regions at distances comparable with this, the resulting absorption of light must play a very important rôle in limiting the apparent extent of the universe. If this absorption is of the type which is produced by dust, or even by particles of the size of the drops of water in ordinary clouds, it will affect all wave-lengths to substantially the same extent, and be much more difficult to detect than the gaseous scattering, increasing for the shorter wave-lengths, which several investigators have sought for, but whose existence Shapley has apparently disproved.[22] It seems appropriate to remark in this connection that absorption independent of the wave length seems *a priori* much more likely to occur than the other, since the same quantity of matter in the form of a fog is incomparably more effective than in gaseous form, (compare the opacity of a few meters of cloud with that of all the rest of the atmosphere) and also since most forms of matter are likely to be in the solid or liquid state at the temperatures prevailing in interstellar space.

(*c*) The forms of nebulae—especially of planetary and ring nebulae—deserve careful study. As Campbell suggests,[73] it is difficult to account for them without assuming the existence of some repulsive force which

counteracts the attraction of the nucleus. He suggests light-pressure—which would fit in well with views of the origin of the luminosity such as are suggested above. In such a case we should anticipate that most of the light of the nebula would come from the nucleus, and this appears to be usually, though not always, the case.

(*d*) Measures of the radial velocities of nebulae have already shown that the planetary nebulae, as a class, are moving in space much more rapidly than the stars;[74] that there exist internal motions within them, usually of a rotational character, but sometimes more complicated;[73] and that, in order to keep the moving material from flying away into space, the total masses of the nebulae must be very considerable, and probably a good deal larger than those of the stars.[73] Much remains to be done in the investigation of these motions, and in their interpretation. The proper motions of planetary nebulae, and perhaps in some cases the internal motions of the nebular material, can be determined by comparison of suitable photographs, and it is probable that in a decade or two we shall obtain in this way a fair idea of the distances and real dimensions of these bodies. Observations for parallax on some of the larger and presumably nearer planetary nebulae are also desirable.

The extended gaseous nebulae should be examined spectrographically to see whether turbulent motions exist in others, as they do in the great nebulae of Orion;[75] and it would be worth while to compare photographs of some of those which show sharp details, in the hope of detecting proper motion, either of the whole or of parts.

Investigations of the distribution within the gaseous nebulae of the substances which give the different spectral lines may be made by photography either with absorbing screens or with slitless spectroscopes, and promise information regarding the conditions prevailing in the nebulae, and the mutual relations of the lines of unknown origin.

14. (*a*) The *spiral nebulae* have been shown by recent investigations to be the most extraordinary objects in the heavens. Their enormous radial velocities—first detected by Slipher[76]—and the almost equally rapid internal motions within them,[77] put them in a class by themselves. Further measures of these motions are needed; and, when the radial velocities of a sufficient number of spirals, well distributed over the heavens, are known, it may be possible to determine definitely the direction and rate of the motion of the Sun (and presumably of the whole galactic system) with respect to the system of nebulae. The provisional determination by Young and Harper,[78] from very scanty data, indicates for the motion of our system the enormous velocity of 600 kilometers per second.

(*b*) As van Maanen[79] and others[80] have shown, the proper motions of some spiral nebulae—both of the mass as a whole and of the condensations in the arms relatively to the centre—are apparently large enough to be determined by the careful comparison of plates taken only a few years apart. This opens up another wide field of study, and will make it possible before long to determine the mean parallax of many such nebulae by comparison of the proper motions and radial velocities of their nuclei. There is also reason to hope that the distances of some individual nebulae, which are seen at a suitable angle, can be determined by comparing the radial and transverse components of motion along the arms. Enough is already known to convince us that the distances of these nebulae must be measured in thousands of parsecs, and their diameters in parsecs, and that direct measures for parallax are utterly hopeless.

(*c*) Photometric measures, both of the total light of the spirals and the relative brightness of their parts, would be of value, especially if accompanied by determinations of color. Seares[81] has recently shown that the outer convolutions are far bluer than the centre—which is the part that shows the spectrum of solar type. Spectroscopic observations of these outer regions, if possible, would be of great interest. Another matter calling for further study is the nature of the dark bands which cross many nebulae which appear to be spirals seen edgewise, and look as if they were due to the interposition of opaque material in the outer regions of the nebula.

(*d*) The distribution of spiral nebulae in the heavens—so utterly different from that of any other objects—may be explainable when their real distribution in space is even partially known. It is hardly time as yet to consider the greater question of their real nature, except to note, with van Maanen,[79] that, unless they are in process of very rapid dissipation into space, their masses must be exceedingly great.

15. Finally, it must not be forgotten how important a place *theoretical investigations* will occupy in the solution of the larger problems of sidereal astronomy. The increasing observational data are already furnishing just those guides which point the skilled mathematician in the right direction, and these indications have been very successfully followed, especially by certain members of that 'Cambridge school' which combines keen mathematical analysis with a thorough knowledge of modern physics. Results of remarkable generality have already been obtained.

In the field of stellar evolution, Eddington[82] has worked out in detail the importance of radiation pressure in determining the conditions of internal equilibrium of the stars, and the approximate equality in

brightness of the giant stars of all spectral types has found a simple explanation.

If the conclusion that the luminosity of a giant star is a function of its mass, but not of its temperature or age, is confirmed, and the nature of the function fixed by observation, the problem of determining the masses of stars which are not double will in many cases be solved.

Jeans,[83] discussing the problem of the figures of equilibrium of a rotating mass of compressible fluid, has already reached conclusions which not only bear upon the origin of double stars, but have suggested an entirely new and very stimulating conception of the nature of spiral nebulae, as huge rotating masses of gas, which, becoming unstable at the edge under the influence of their own rotation and the attraction of the neighboring stars, throw off matter from their periphery in streams of such enormous size that they may divide into 'nuclei' large enough to form ordinary stars upon condensation.

In the field of galactic astronomy, Schwarzschild[84] has developed powerful methods for handling the statistical material which must be our main guide, and Jeans[85] and Eddington[86] have shown that 'star streaming' demands no unknown forces for its explanation, but is probably interpretable dynamically, as a property of a system of stars in motion under their own gravitation—although the existence of 'streaming' appears to indicate that the galactic system is not in a 'steady state.' Eddington[87] has shown that the similarity of distribution of the stars in different globular clusters presents a problem by no means simple, though of much interest.

Almost the whole of this work has appeared within the last three years, and further notable advances may be anticipated. Indeed, almost as these words are written, comes the first installment of an important paper by Eddington[88] on the oscillations of a gaseous star, which may afford the long-sought solution of the problem of Cepheid variation.

Among other specific problems awaiting discussion may be mentioned the question whether the tidal interaction of two compressible and slowly condensing bodies can cause an originally small eccentricity to increase to the very large values which are found in many visual binaries, and some spectroscopic binaries as well; and, if this proves to be impossible, how the systems in question can have originated;[89] the origin and laws of the complicated changes which occur in the periods of many eclipsing binaries; and the equilibrium and motions of the constituent parts of planetary and spiral nebulae.

Mention should also be made of the work of Nicholson[93] on the interpretation of unknown lines in the spectra of nebulae and of the

solar corona as arising from hypothetical atoms of very simple structure —which has successfully met the test of prediction—and of the development of the theory of general relativity, which has already been used by deSitter[94] to set a superior limit to the whole quantity of matter in the universe, and may have important applications in future.

16. Of more fundamental nature, and obvious importance, is the unsolved problem of the source of the energy which the stars are continually radiating at so rapid a rate. It is becoming increasingly plain that the gravitational energy liberated by contraction from infinity would not nearly suffice to maintain the Sun's radiation during geological time[90] (according to even the more conservative estimates of the latter); yet the mere continuous existence of life on the Earth is evidence that the Sun has not merely kept on shining throughout this interval, but has not changed in brightness by more than one magnitude, at the outside. In the case of some giant stars, contraction from infinity would hardly suffice to furnish the energy which they have radiated during historic time.[91] There appear to be two ways out of the difficulty; either the stars do not radiate heat in all directions to space at the same rate as they do towards the Earth, or else they have some unknown and exceedingly great supplies of internal energy. The first alternative, however, seems to be excluded by the fact that the amount of heat which the Earth receives from the Sun, and loses again by radiation into space, is not greatly, and probably not at all, inferior to that which a black body of the same size and temperature as the Earth's effective radiating surface would radiate to an enclosure at the absolute zero.[92] There seems therefore no escape from the conclusion that the heat radiated by a star can not be provided by contraction. What the source of the energy may be, how it is converted into heat in the body of the star, and where it goes after passing from the star's surface into the ether, are at present the greatest of all the unsolved problems of astronomy.

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